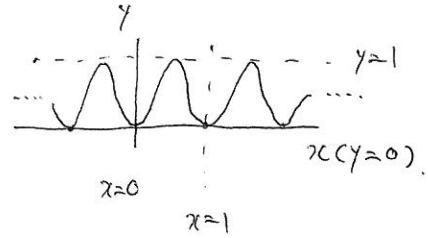
* 2021학년도 대수능 수학 가형 30번.

$f(x) = x^3 + \cdots$, $g(x) = f(\sin^2 \pi x)$.

$\sin^2 \pi x \to$



(가) $(0<x<1)$ $g(x)$의 극대는 세 곳, 극댓값은 동일(1개)

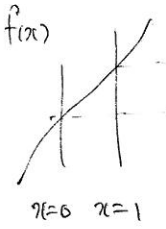
(나) $(-\infty, \infty)$ $0 \le g(x) \le \frac{1}{2}$.

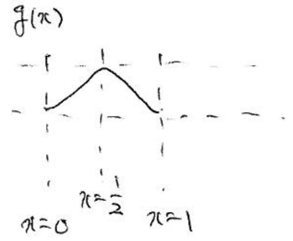
$(0(+),\ 1(-))$ $\sin^2 \pi x$는

$0(+) \to 1 \to 0(+)$

(증가) (감소).

→ $f(x)$가 극대, 극소가 없는 경우라면

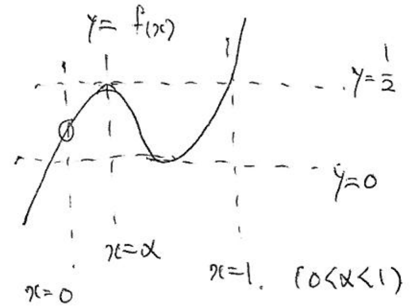




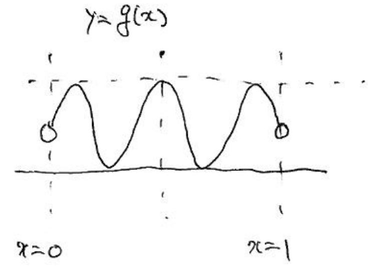
→ 극대는 한 곳 뿐. ∴ $f(x)$는 극대, 극소가 모두 존재해야 한다.

($f(x)$가 $x=\alpha$에서 극대라고 하면)

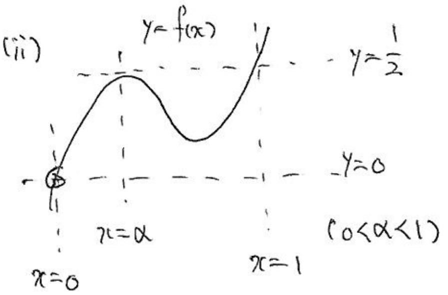
(i)



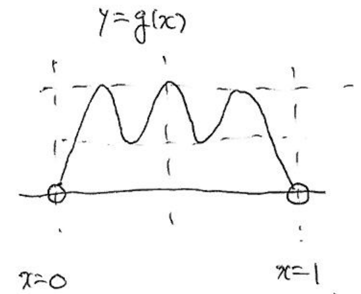
→



(ii)



→



$f'\left(\frac{2+\alpha}{3}\right) = 0$.

$g'(x) = f'(\sin^2 \pi x) \cdot 2\sin \pi x \cdot \cos \pi x \cdot \pi$

$g'\left(\frac{1}{2}\right) = 0$.

$g\left(\frac{1}{2}\right) = f(1) = \frac{1}{2}$

$\therefore f(x) = (x-\alpha)^2(x-1) + \frac{1}{2}$, $f'(x) = 2(x-\alpha)(x-1) + (x-\alpha)^2$

$= (x-\alpha)(2x-2+x-\alpha) = 3(x-\alpha)\left(x - \frac{(2+\alpha)}{3}\right)$

(i) $f\left(\frac{2+\alpha}{3}\right) = 0 = \left(\frac{2-2\alpha}{3}\right)^2\left(\frac{\alpha-1}{3}\right) + \frac{1}{2}$

$= \frac{4}{27}(\alpha-1)^3 + \frac{1}{2} = 0$.

$\therefore (\alpha-1)^3 = -\frac{27}{8}$에서 $\alpha - 1 = -\frac{3}{2}$.

$\alpha = -\frac{1}{2}$. $(0<\alpha<1)$ 조건에 위배.

(ii) $\lim_{x \to 0+} g(x) = \lim_{x \to 0+} f(\sin^2 \pi x) = \lim_{x \to 0+} f(x)$.

$\therefore f(0+) = -\alpha^2 + \frac{1}{2} = 0$. $\therefore \alpha = \frac{\sqrt{2}}{2}$ $(0<\alpha<1)$.

따라서 $f(x) = \left(x - \frac{\sqrt{2}}{2}\right)^2(x-1) + \frac{1}{2}$.

$\therefore f(2) = 4 + \frac{1}{2} - 2\sqrt{2} + \frac{1}{2} = 5 - 2\sqrt{2}$ //

* 2021학년도 대수능 수학 나형 30번.

$f(x) = x^3 + \cdots$, $g(x)$는 일차함수

$$h(x) = \begin{cases} |f(x) - g(x)| & (x<1) \\ f(x) + g(x) & (x \ge 1) \end{cases}$$

$h(x)$ 미분가능.

$h(0) = 0$, $h(2) = 5$. $h(4) = ?$

① $h(0) = 0$. 이기 $\rightarrow$ $f(0) - g(0) = 0$.

② $\rightarrow$ $f'(0) - g'(0) = 0$.

$|\Delta(x)|$ 가 $\Delta(x_1) = 0$일 때 미분가능하려면 $\Delta'(x_1) = 0$ 이어야 한다.

③ $x=1$에서 연속. $|f(1-) - g(1-)| = |f(1) - g(1)| = f(1) + g(1)$.

④ $h(2) = 5$. $f(2) + g(2) = 5$.

(i) ③에서 $f(1) > g(1)$ $\rightarrow$ $\begin{cases} f(1) - g(1) = f(1) + g(1). & \therefore g(1) = 0 \\ f'(1) - g'(1) = f'(1) + g'(1). & \therefore g'(1) = 0 \end{cases}$ $\rightarrow$ 조건 위배.

(상수함수 ≠ 일차함수. ∴ $f(1) = g(1)$에서도 성립 ×)

$f(1) < g(1)$ $\rightarrow$ $\begin{cases} -f(1) + g(1) = f(1) + g(1). & \therefore f(1) = 0 \\ -f'(1) + g'(1) = f'(1) + g'(1) & \therefore f'(1) = 0 \end{cases}$

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$g(x) = px + q$ $(p \neq 0)$

$g'(x) = p$

① $f(0) = g(0)$. $\therefore c = q$

② $f'(0) = g'(0)$. $\therefore b = p$.

③ $f(1) = 1 + a + b + c = 0$

③. $f'(1) = 3 + 2a + b = 0$.

④ $h(2) = f(2) + g(2) = 8 + 4a + 2b + c + 2p + q = 4a + 4b + 2c + 8 = 5$.

$\therefore$ $a + b + c = -1$

$2a + b = -3$

$4a + 4b + 2c = -3$.

$c = -\frac{1}{2}$. $\therefore q = -\frac{1}{2}$. $(a + b = -\frac{1}{2})$. $\therefore a = -\frac{5}{2}$. $b = 2$. $p = 2$.

$f(x) = x^3 - \frac{5}{2}x^2 + 2x - \frac{1}{2} = (x-1)^2(x - \frac{1}{2})$

$g(x) = 2x - \frac{1}{2}$

$h(4) = f(4) + g(4) = 9 \times \frac{7}{2} + 8 - \frac{1}{2} = \frac{62}{2} + 8 = 31 + 8 = 39$ //